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Note on Mr. Browning's Automatic Spectroscope.

By Richard A. Proctor, B.A.

It occurred to me, while Mr. Browning's spectroscope was under discussion at the last meeting of the Society (and more particularly when Prof. Pritchard raised the question whether all rays pass through the battery with minimum deviation for each prism), that by a slight modification Mr. Browning's ingenious device was capable of more completely fulfilling the conditions of the problem suggested by Prof. Pritchard. I am not sure that practically the spectrum will be viewed under much more favourable conditions by the plan I am about to propose; but fortunately the plan has (Mr. Browning tells me) other advantages. I need hardly say that I only present it as a modification of his most ingenious plan, and claim no separate merit for the new arrangement, even though the optical performance of the battery of prisms should be appreciably improved by it.

It is necessary to premise that although it is possible, as I shall show, to secure minimum deviation for rays of all degrees of refrangibility, yet we can obtain only an approximation to that condition which minimum deviation is intended to secure. Even in the case of a single prism, the primary and secondary foci of the emergent pencil corresponding to rays of given refrangibility are not absolutely coincident when such rays pass through the prism with minimum deviation. If ϕ and ψ' represent the angles of incidence and emergence, ϕ' and ψ the angles of first and second refraction, we have for the distance between the primary and secondary foci of the emergent pencil the expression

$$\left[\frac{\cos^2\phi'\cos^2\psi}{\cos^2\phi\cos^2\psi'}\,u + \frac{t\cos^2\psi}{\mu\cos^2\psi'}\right] - \left[u + \frac{t}{\mu}\right]$$

where u represents the distance of the origin of the source of light from the point of incidence, t the length of the path within the prism, and μ the refractive index of the prism for the rays considered. Hence, that the primary and secondary foci of the emergent pencil may coincide, we must have

$$(\cos^2\phi'\cos^2\psi - \cos^2\phi\cos^2\psi')\,\mu\,u = t\cos^2\phi\,(\cos^2\psi' - \cos^2\psi).$$

Only when t vanishes does this reduce to the condition

$$\cos^2\phi'\cos^2\psi = \cos^2\phi\cos^2\psi'$$

or

$$\phi = \psi',$$

which is the condition of minimum deviation.

Now t is ordinarily of appreciable length for each prism, in the case of a battery of prisms; and therefore the resulting distance between the primary and secondary foci is also appreciable, though small, even in the case of the first prism of the battery,

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and will increase with each successive prism. The expression for this distance reduces, in the case of minimum deviation, to

$$\frac{t}{\mu}(1-\mu^2)\tan^2\psi',$$

or, if the refracting angle of the prism be $2i$, to

$$\frac{t}{\mu}(1-\mu^2)\tan^2 i.$$

The circle of least confusion will therefore have a radius represented by

$$\frac{t\lambda(1-\mu^2)\tan^2 i}{\mu u\left[\sqrt{1-\mu^2\sin^2 i}+1\right]+t\left[(1-\mu^2\sin^2 i)^{\frac{3}{2}}+1\right]}$$

where λ is the breadth of the pencil at the place of emergence. It is obvious, however, that this expression will be minute under the ordinary conditions of spectroscopic observation.

Setting the condition of minimum deviation, instead of the true optical condition, as that which we are to aim at, it is obvious that in the case of a battery of prisms, all of equal refracting angle, what we require is that, first, the bases of all the prisms shall always circumscribe a circle; and, secondly, that the angle of incidence on the first surface of the first prism shall be equal to the angle of emergence from the second surface of the last prism. If these conditions are fulfilled, the condition for minimum deviation must necessarily be fulfilled; if they are not fulfilled, there cannot be minimum deviation.

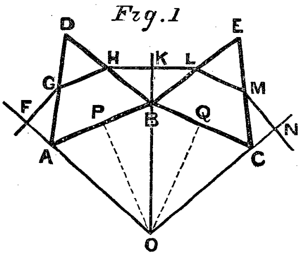
Fig. 1

Suppose, for instance, that A D B and B E C (fig. 1) are two prisms of equal refracting angles, D and E, and connected, as in Browning's spectroscope, at the angle B; and let F G H L M N be the path of a ray refracted with minimum deviation through each. Then

$$\cos BHK = \mu\sin\frac{D}{2} = \mu\sin\frac{E}{2} = \cos BLH;$$

that is, B H L is an isosceles triangle, and O B K, the bisector of the angle D B E, is perpendicular to H L.

Now P O and Q O, the rectangular bisectors of A B and B C, meet on B O, as at O; and O A F makes with G A the angle G A F, equal to the angle H B K. But the angle A G F is equal to the angle B H K. Hence the angle A F G is equal to the angle B K H, or is a right angle.

In like manner O C N is at right angles to M N.

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If another like prism be added at C, the same reasoning will apply; and so on, whatever be the number of prisms.

It follows that the lines from a single point O to the attached angles A, B, C, &c. of any such series of prisms are perpendicular to the course of the ray between the prisms. But any two perpendiculars O P and O Q from this point O on successive base-lines A B and B C are obviously equal; hence all such perpendiculars are equal; and therefore A B, B C, &c. are all tangents to a circle about O as centre. Further

$$\angle\, FGA = \angle\, BHK = \angle\, BLH;$$

that is, the angle of incidence on any prism of the series is equal to the angle of incidence on the next. Therefore the angle of incidence on the first prism is equal to the angle of incidence on the last, and therefore to the angle of emergence from the last.

These are the conditions, then, which must be fulfilled in all positions of a self-adjusting battery, if rays of all degrees of refrangibility are to pass with minimum deviation through each prism, viz.

1st. *All the bases must be tangents of a circle.*

2dly. *The pencil must emerge from the last prism at the same angle at which it fell on the first prism.*

The first condition Mr. Browning's plan obviously fulfils, for the perpendiculars bisecting the equal bases all pass through one point.

The second condition the plan does not, as first devised, accurately fulfil, because the first prism is fixed, and the incident white light falls on it from the slit at a constant angle; so that there can be minimum deviation only for rays of a certain degree of refrangibility.*

The modification I suggest consists merely in fixing an angle (the angle corresponding to A in fig. 1) of the first prism, and causing this prism to move so as that the line corresponding to O A F shall be constantly perpendicular to the axis of the incident pencil F G. Since the axis of the incident pencil is fixed in position, this involves only the addition of a fixed slot in the position O A F.

It further occurred to me that since by this arrangement the axis of the emergent pencil from the last prism is situated (as M N in fig. 1) in a direction at right angles to the line from O to the angle corresponding to C, all that is necessary to keep the viewing telescope in its true position, whatever part of the spectrum is studied, is that a slot coincident in direction with this

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* Thus, if the constant angle of incidence be ϕ, and the refracting angle of each prism $2\,i$, minimum deviation is secured only for rays whose refractive index is $\frac{\sin \phi}{\sin i}$.

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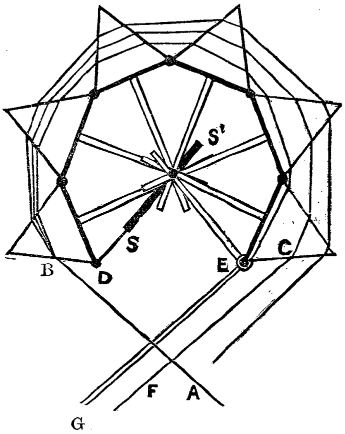
last-named line should be rigidly attached to an arm bearing the telescope, and should be also at right angles to the optical axis of the telescope.

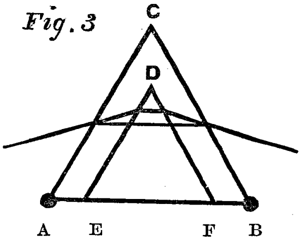
Fig 2



In fig. 2 the actual position of the prisms is indicated. A B is the axis of the incident pencil at right angles to the fixed slot S S′ (whose direction passes through the angle D of the first prism). The double lines indicate the position of the movable slots, the last of which, O E, is rigidly attached to the viewing telescope, and at right angles to its axis, which corresponds in position with C F, the axis of the emergent pencil.

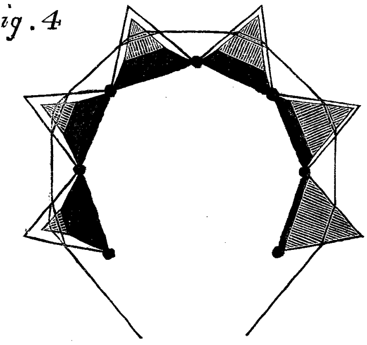
To obtain the greatest possible dispersion, without losing any part of the spectrum, the refracting angles of the prisms should of course be such that E G will just touch the angle D when the extreme limit of the visible spectrum at its most refrangible end is in the field.

Fig. 3



I may add that if A B (fig. 3) be the base of any one of the prisms of the series, it will be evident from the figure that the prism A C B may be replaced by a prism E D F (A E being equal to F B, and E D, F D parallel respectively to A C, B C) without affecting the subsequent path of the *axis* of any pencil; and that for certain purposes the shortening of the path within the prism by such a change would be advantageous. I need not enter into the consideration of the other ways in which a similar shortening can be effected; but I may note that there seems nothing to prevent the equal prisms used in Mr. Browning's present arrangement from being replaced by a series of increasing prisms, represented in section by the shaded triangles in fig. 4 (the last prism of the battery remaining unchanged). If this arrangement is practically feasible, the definition would, I imagine, be appreciably improved. As the figure shows, the course of the axis of a pencil would not be so altered as to affect the path between the prisms, or the equality of the angles of incidence and emergence.

Fig. 4



Since the above was written, I have devised a plan by which

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minimum deviation can be secured automatically for a battery of nine, ten, or eleven equilateral prisms. The light from the sixth prism of a series like the above (figs. 2 and 4) is received on a totally reflecting prism, from which it passes to a second battery in a reversed position. By a very simple contrivance the totally reflecting prism adjusts itself, so as always to hold a symmetrical position between the two batteries, the faces including the right-angle of the prism being always perpendicular to the light passing out of one battery and into the other. But as a long connexion, such as this arrangement involves, necessarily works imperfectly (however true theoretically), I at once double the dispersive power and halve the effective length of the connexion, by causing the light to be reflected so as to re-traverse the whole battery, and to be viewed by a fixed telescope whose optical axis is in part coincident with that of the collimator. The motion of adjustment thus has to be communicated to the totally reflecting prism, and the effective length of connexion is the same as for a single battery. I hope to be able to describe this plan in full at the next meeting of this Society. It need hardly be said that the arrangement is only useful where there is abundance of light. One secures by means of it a dispersion corresponding to that given by 22 or 23 equilateral prisms (according to the way in which the light is sent back through the battery); but the power can be reduced prism by prism to any extent which may be requisite.

The Satellites of Uranus. By Richard A. Proctor, B.A.

It has been with extreme regret that I have seen a passage in my *Other Worlds* interpreted by a Vice-President of this Society as a setting up of my mere opinion against the telescopic observations of our President. I hasten to correct this misinterpretation. In expressing the opinion that Sir William Herschel was not deceived as respects the four satellites of *Uranus* which remain unidentified, I was but doing what Admiral Smyth, Professor Nichol, and Professor Grant, had already done (simply on the score of Sir William Herschel's wonderful accuracy as an observer), I am convinced that Mr. Lassell (whose name, by the bye, I had not mentioned in the matter) would be the last to condemn this confidence in the work of an astronomer whose achievements he has so worthily emulated.

I wrote, it is true, with full knowledge that Mr. Lassell has not detected other satellites than *Umbriel*, *Ariel*, *Oberon*, and *Titania;* and with full knowledge also of the qualities of his four-feet mirror. But Sir William Herschel's observations indicate a variability in the lustre of the satellites quite sufficient to account for Mr. Lassell's want of success (so far): and though the light-gathering power of Herschel's four-feet mirror was doubtless inferior to that of Mr. Lassell's, yet it must be remem-